

# Entangling Atoms and Ions in Dissipative Environments

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Quantum information processing rests on our ability to manipulate quantum superpositions through coherent unitary transformations, and to establish entanglement between constituent quantum components of the processor. The quantum information processor (a linear ion trap, or a cavity confining the radiation field for example) exists in a dissipative environment. We discuss ways in which entanglement can be established within such dissipative environments. We can even make use of a strong interaction of the system with its environment to produce entanglement in a controlled way.

## I. INTRODUCTION

In quantum information theory, information is stored in the states of a quantum mechanical system instead of in the bits of a classical computer. One possible system to realise a single quantum mechanical bit (qubit) is given by a two-level atom with the states  $|0\rangle$  and  $|1\rangle$ . The atoms can be stored for instance in a linear trap, an optical lattice and/or inside an optical cavity. The main advantage of using quantum mechanical systems arises from the fact that the system can be prepared in a superposition of states. But to do so, one has to be able to prepare the system in any arbitrary initial state. These states are in general entangled. In addition, one has to find ways to manipulate the state of the system in a controlled way.

The main problem in the realisation of quantum computing or quantum information processing is decoherence [1,2]. The reason for this is that it is difficult to isolate a quantum mechanical system from its environment. For instance in the case of two-level atoms, spontaneous emission leads to the loss of information. Therefore, decoherence is usually thought of as an entirely destructive effect. Quantum error correction codes have been invented [3] and the use of decoherence-free subspaces [2,4–6] for quantum computing has been proposed.

Here we go a step further. In contrast to the widely held folk-belief of decay being something entirely negative, we demonstrate how decay can lead to entanglement of quantum systems rather than destroying entanglement. Indeed, the detection of decay (or no decay) can be used as a method of state preparation, and can allow a wide range of quantum computation and communication tasks [7–11]. In this paper we give examples in which the interaction between a system and its environment is used to create entanglement of a system, rather than destroying it. In Section II and III we discuss the preparation and manipulation of entangled states of atoms inside a cavity. In Section IV we show how quantum communication protocols such as teleportation can be implemented by the detection of photon decay from cavities containing trapped ions or atoms.

## II. ENTANGLING TWO ATOMS INSIDE A CAVITY

First we give an example of how two atoms can be prepared in a maximally entangled state by making use of the interaction of the system with its environment. We will assume a strong coupling between the system and the surrounding free radiation field. To prepare the system in a pure state we make use of the time evolution under the condition that no photon emissions are observed. The scheme is very simple and should be easy to implement with present technology. Its success rate is about 50 %. Using similar ideas, but exploiting the adiabatic elimination of fast variables in a decaying system, even higher success can be achieved. This will be discussed in the following Section.

### A. The physical system

Our system consists of two identical two-level atoms/ions confined in a linear trap or an optical lattice as shown in Fig. 1. The atoms are surrounded by an optical cavity. The energy eigenstates of the atoms are denoted by  $|0\rangle_i$  and  $|1\rangle_i$  ( $i = 1, 2$ ) in the following. We assume that the distance between the atoms is much larger than an

optical wavelength which allows us to assume that each atom can be individually addressed with a laser pulse. The atomic transition is assumed to be in resonance with the cavity mode and its frequency  $\omega_0$  equals the frequency of the resonator mode  $\omega_{\text{cav}}$ . Each atom couples to the single field mode inside the cavity with a coupling constant  $g_i$ . For simplicity we assume  $g_1 = g_2 \equiv g$ . The relaxation of the ion-cavity system can take place through two different channels. Each atom can spontaneously emit a photon with a rate  $\Gamma$ . In addition, a photon inside the cavity mode leaks out through the cavity mirrors with a rate  $\kappa$ . Note the presence of a single photon detector  $D$  in our scheme. This set up will allow us to monitor photons leaking through the cavity mirrors. The detection efficiency varies with the wavelength but it can be up to 90%.

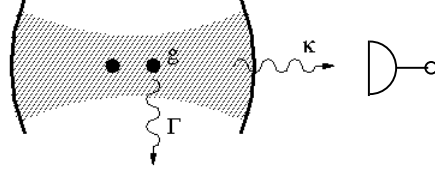


FIG. 1. Experimental set-up. The system consists of two two-level atoms placed inside a leaky cavity. The decay rate  $\Gamma$  describes the spontaneous emission of each atom, while the rate  $\kappa$  refers to photons leaking through the cavity mirrors. The latter can be monitored by a detector.

The field annihilation operator for the cavity mode is denoted by  $b$  in the following. In addition the atoms are weakly coupled to the free radiation field outside the cavity with a coupling constant  $g_{\mathbf{k}\lambda}^{(i)}$  for the  $i$ th atom and a field mode with wave vector  $\mathbf{k}$  and polarisation  $\lambda$ . The annihilation operator for this mode is  $a_{\mathbf{k}\lambda}$ . This free radiation field provides a “heat bath” for the atoms and is responsible for spontaneous emission. We take also into account non-ideal cavity mirrors by coupling the field inside the resonator to the outside with a strength  $s_{\mathbf{k}\lambda}$ , so that single photons can leak out. The annihilation operator of the free radiation field to which the cavity field couples is given by  $\tilde{a}_{\mathbf{k}\lambda}$ . Then, in the Schrödinger picture the Hamiltonian of the system and its environment is given by

$$H = \sum_{i=1,2} \hbar\omega_0 \sigma_i^\dagger \sigma_i + \hbar\omega_0 b^\dagger b + \sum_{\mathbf{k}\lambda} \hbar\omega_k \left( a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + \tilde{a}_{\mathbf{k}\lambda}^\dagger \tilde{a}_{\mathbf{k}\lambda} \right) + i\hbar \sum_{i=1,2} g b \sigma_i^\dagger + i\hbar \sum_{i=1,2} \sum_{\mathbf{k}\lambda} g_{\mathbf{k}\lambda}^{(i)} a_{\mathbf{k}\lambda} \sigma_i^\dagger + i\hbar \sum_{\mathbf{k}\lambda} s_{\mathbf{k}\lambda} \tilde{a}_{\mathbf{k}\lambda} b^\dagger + \text{h.c.} \quad (1)$$

The first four terms give the interaction free Hamiltonian and correspond to the free energy of the atoms, the resonant cavity mode, and the electromagnetic fields outside the system. Going over to the interaction picture with respect to the interaction free Hamiltonian gives rise to the interaction Hamiltonian

$$H_I = i\hbar \sum_{i=1,2} g b \sigma_i^\dagger + i\hbar \sum_{i=1,2} \sum_{\mathbf{k}\lambda} g_{\mathbf{k}\lambda}^{(i)} a_{\mathbf{k}\lambda} \sigma_i^\dagger e^{i(\omega_0 - \omega_k)t} + i\hbar \sum_{\mathbf{k}\lambda} s_{\mathbf{k}\lambda} \tilde{a}_{\mathbf{k}\lambda} b^\dagger e^{i(\omega_0 - \omega_k)t} + \text{h.c.} \quad (2)$$

The first term contains the coupling of the atoms to the cavity mode. The second term describes the coupling of the atoms to the free radiation field and is responsible for spontaneous emission. The last term describes the damping of the cavity mode by leaking of photons through the cavity mirrors.

## B. A preparation scheme

In the following we assume that the parameters in the proposed experiment fulfill the condition

$$\Gamma \ll \kappa, \quad g^2/\kappa. \quad (3)$$

Using this parameter regime a maximally entangled state of the atoms can be prepared in the following way. We assume that both atoms are initially in the ground state while the cavity mode is empty. Then atom 1 is driven into the excited state by application of a laser pulse with the length of a  $\pi$  pulse. In this way we introduce an excitation into the system and the initial conditions for our scheme will be given by the composite state

$$|\psi_0\rangle = |0\rangle_{\text{cav}} \otimes |1\rangle_1 \otimes |0\rangle_2 \equiv |010\rangle. \quad (4)$$

After this  $\pi$  pulse, during a short waiting time

$$\Delta t \gg 1/\kappa, \kappa/g^2 \quad (5)$$

the system may emit a photon. Whether a photon emission takes place or not has to be measured by the detector  $D$  shown in Fig. 1. Depending on the outcome of this measurement the preparation scheme succeeds or fails and has to be repeated again.

If the system emits a photon its excitation is lost and the state vector is restored to the ground state  $|000\rangle$ , the initial state of the scheme. But there is also the possibility that the excitation remains inside the system. The probability for this to happen equals 50 % and coincides with the success rate of the scheme we propose. In this case the atoms are prepared in the state

$$|\psi\rangle = |0a\rangle \quad \text{with} \quad |a\rangle \equiv (|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)/\sqrt{2} , \quad (6)$$

which is a maximally entangled state of the two atoms. We will show later that the prepared state  $|0a\rangle$  is stable and does not change in time as long as no additional interaction is applied.

### C. An intuitive explanation why the scheme works

First we give simple arguments to show why the preparation scheme works. The main underlying reason is given by the fact that the atomic states  $|0\rangle_1|0\rangle_2$  and  $|a\rangle$  are *trapped* states [8,12–14]. If the cavity mode is empty no interaction between the mode and the atoms takes place, which can be proven easily with the help of the first term on the right hand side of Eq. (2). Applying this operator from Eq. (2) onto a state of the general form

$$|\psi\rangle = \alpha |000\rangle + \beta |0a\rangle \quad (7)$$

always gives zero. Even if the atoms are excited, they cannot transfer their excitation into the cavity. No photon leaks out through the cavity mirrors. As a consequence, a system once prepared in a state with an empty cavity field and a trapped state of the atoms does not change in time. The states (7) are the *decoherence-free* states of the system [9,11]. On the other side, if the system is prepared in a state with no overlap with  $|\psi\rangle$  given in Eq. (7) it will emit a photon during the time  $\Delta t$  for sure. The observation of the system over a time interval  $\Delta t$  can be interpreted as a measurement of whether the system is decoherence-free or not.

Making now use of the projection postulate [15] the measurement theory suggests the following: If the initial state of the system is given by  $|\psi_0\rangle = |010\rangle$  the system is at the end of the measurement in case of no photon emission given by  $|0a\rangle$ . The probability for no photon emission (which coincides with the success rate of the scheme) equals the overlap of  $|\psi\rangle$  with the decoherence-free subspace given by Eq. (7),

$$P_0(\Delta t, \psi_0) = |\langle 000|\psi_0\rangle|^2 + |\langle 0a|\psi_0\rangle|^2 = \frac{1}{2} . \quad (8)$$

If the initial state  $|\psi_0\rangle$  of the system is a product state with no population in state  $|000\rangle$ , this factor of one half is the highest success rate in preparing a maximally entangled state of both atoms which can be obtained by the scheme proposed here.

Summarising, we need the interaction between the atoms and the cavity mode to be assured that the prepared state is an entangled state. The non-Hermitian term in the no-jump evolution describing the decay of unstable states was used to remove the unwanted states during the no-photon time evolution. To make use of the no-photon case is, in general, essential because the system is otherwise prepared in a statistical mixture of states.

### D. Results of a detailed analysis

The time evolution of the system under the condition of no photon emissions can easily be described using the quantum jump approach [16,17], with the help of the relevant conditional Hamiltonian  $H_{\text{cond}}$ . This Hamiltonian is non-Hermitian and leads to a non-unitary time evolution operator  $U_{\text{cond}}(t, 0)$ . The norm of a state vector developing through  $U_{\text{cond}}$ ,

$$|\psi^0(t)\rangle = U_{\text{cond}}(t, 0) |\psi_0(t)\rangle , \quad (9)$$

decreases in time and can be used to determine the probability to find no photons in  $(0, t)$ , if the system was initially prepared in the state  $|\psi_0\rangle$ ,

$$P_0(t, \psi_0) = \| U_{\text{cond}}(t, 0) |\psi_0\rangle \|^2 . \quad (10)$$

The state of the system under the condition of no photon emission at time  $t$  equals the normalised state of Eq. (9) which still has to be normalised. The conditional Hamiltonian can be derived from the Hamiltonian  $H_I$  of Eq. (2) [16]. In the case of the system considered here we find [8]

$$H_{\text{cond}} = i\hbar g \sum_{i=1,2} \left( b\sigma_i^\dagger - \text{h.c.} \right) - i\hbar \Gamma \sum_{i=1,2} \sigma_i^\dagger \sigma_i - i\hbar \kappa b^\dagger b . \quad (11)$$

The first term describes as above the interaction between the atoms and the cavity mode. The last two terms lead to a decrease of the norm of a state vector if photons can be emitted or leak out through the cavity mirrors.

To describe the system we only need to consider a three dimensional subspace formed by the states  $|001\rangle$ ,  $|010\rangle$ , and  $|100\rangle$ , because the energy in the system does not change here during the conditional time evolution. In this subspace the conditional time evolution operator can be written as

$$U_{\text{cond}}(t, 0) = \sum_{j=1,2,3} e^{-i\lambda_j t/\hbar} |\lambda_j\rangle \langle \lambda^j| , \quad (12)$$

where  $\lambda_j$  are the eigenstates of  $H_{\text{cond}}$ ,  $|\lambda_j\rangle$  the corresponding eigenstates and the  $\langle \lambda^j|$  are defined by the relation  $\langle \lambda^j | \lambda_k \rangle = \delta_{jk}$  ( $j, k = 1, 2, 3$ ). Due to the existence of the trapped state  $|0a\rangle$  one of the eigenvalues, let us say  $\lambda_1$ , equals zero and we have  $|\lambda_1\rangle = |0a\rangle$  and  $\langle \lambda^1| = \langle 0a|$ . The other two eigenvalues are given by [8]

$$\lambda_{2/3} = \frac{\hbar}{2i} \left( \kappa + \Gamma \pm i\sqrt{8g^2 + (\kappa - \Gamma)^2} \right) . \quad (13)$$

Using this and Eq. (3) and (5) leads to

$$U_{\text{cond}}(\Delta t, 0) = |0a\rangle \langle 0a| , \quad (14)$$

which is why the conditions on the parameters have been chosen. With the help of Eq. (14) it can now be shown that the scheme works as predicted in Sections II.B. Fig. 2 shows the time evolution of the system under the condition of no photon emission as a function of time assuming the initial state  $|\psi_0\rangle$  of Eq. (4), and has been obtained from a numerical solution of the Eq. (9). Already after a short time  $15/g$  the system has reached the decoherence-free state  $|0a\rangle$  and does not subsequently change in time.

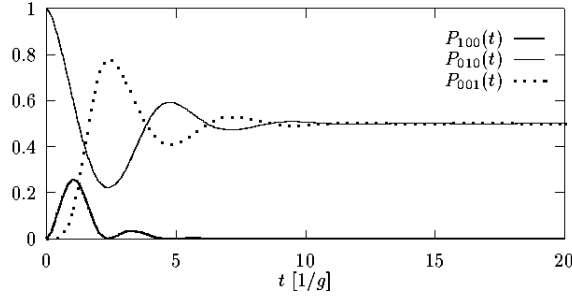


FIG. 2. The time dependence of the probability amplitudes for the basis states  $|100\rangle$ ,  $|010\rangle$  and  $|001\rangle$  under the conditional time evolution that no photon has been detected at all. We have chosen  $g = \kappa$  and  $\Gamma = 10^{-3}g$ . After a short time the cavity mode has decayed and the atoms have reached the pure entangled atomic state.

### III. THE DECOHERENCE-FREE STATES OF $N$ ATOMS AND THEIR MANIPULATION

Decoherence-free states, which are in general entangled states and stable in time as long as no additional interaction is applied, do also exist, if there are more than two atoms inside the cavity. We show in the following that by generalising the scheme discussed in the previous Section to the case of  $N$  atoms decoherence-free subspaces of much higher dimensions can be found. In addition we introduce a new scheme based on an environment induced quantum Zeno effect [18] to manipulate the states *inside* the decoherence-free subspace [9,11,19,20]. The success rate of this new scheme can, at least in principle, be arbitrarily close to 1.

### A. The physical system

Now we assume that the system consists of  $N$  two-level atoms inside a cavity as shown in Fig. 3. Again the atoms are assumed to be in resonance with the single field mode. For simplicity we assume that the coupling of all atoms to the mode is the same and we use the same notation as in the previous Section. It will become clear below that there is no necessity for actual detectors which measure the outgoing photons.

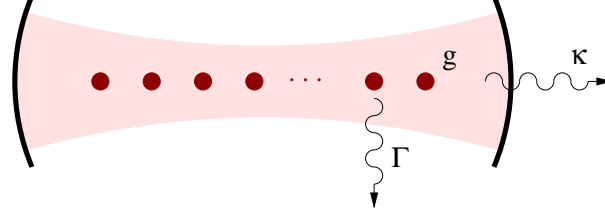


FIG. 3. Schematic view of the system. The two-level atoms are held at fixed positions in the cavity sufficiently far apart that they can be addressed individually by laser beams.

By analogy with Eq. (2) the Hamiltonian of this system is given by

$$H_I = i\hbar \sum_{i=1}^N g b \sigma_i^\dagger + i\hbar \sum_{i=1}^N \sum_{\mathbf{k}\lambda} g_{\mathbf{k}\lambda}^{(i)} a_{\mathbf{k}\lambda} \sigma_i^\dagger e^{i(\omega_0 - \omega_{\mathbf{k}})t} + i\hbar \sum_{\mathbf{k}\lambda} s_{\mathbf{k}\lambda} \tilde{a}_{\mathbf{k}\lambda} b^\dagger e^{i(\omega_0 - \omega_{\mathbf{k}})t} + \text{h.c.} \quad (15)$$

In the same way as for two atoms, the conditional Hamiltonian can be derived from this and one finds

$$H_{\text{cond}} = i\hbar g \sum_{i=1}^N \left( b \sigma_i^\dagger - \text{h.c.} \right) - i\hbar \Gamma \sum_{i=1}^N \sigma_i^\dagger \sigma_i - i\hbar \kappa b^\dagger b. \quad (16)$$

It describes the time evolution of the system as long as no photon is emitted. The two sources of decoherence in the system are again the spontaneous emission of a photon by each atom with a rate  $\Gamma$  and the leakage of a photon through the cavity mirrors with a rate  $\kappa$ . As in the previous Section we assume that the parameters  $\Gamma$ ,  $g$  and  $\kappa$  fulfill condition (3).

### B. Construction of the decoherence-free states

With the help of the quantum jump approach a necessary and sufficient criterion for a state  $|\psi\rangle$  to be decoherence-free can easily be found [6,9]. The probability for the emission of a photon in  $(0, t)$  should be equal to zero if the system is once prepared in a decoherence free state, e.g.

$$P_0(t, \psi) \equiv 1 \quad \forall |\psi\rangle \in \text{DFS}, \quad \forall t. \quad (17)$$

Because of condition (3) we can neglect spontaneous emission ( $\Gamma = 0$ ) in a very good approximation. In the following  $|\mathbf{n}\rangle_{\text{cav}}$  denotes the state with  $n$  photons in the cavity and we define again  $|\mathbf{n}\varphi\rangle \equiv |\mathbf{n}\rangle_{\text{cav}} \otimes |\varphi\rangle$ . To fulfill condition (17) the probability density  $-dP_0(t, \psi)/dt|_{t=0}$  has to vanish. Using Eq. (9) and (10), this leads to  $\langle \psi | (H_{\text{cond}} - H_{\text{cond}}^\dagger) | \psi \rangle = 0$ , and Eq. (16) gives  $|\psi\rangle = |0\varphi\rangle$ . Only if the cavity mode is empty no photon can leak out through the cavity mirrors and the system does effectively not interact with its environment. However, this is not yet sufficient – the cavity mode must *never* become populated. Thus, a state  $|0\varphi\rangle$  is only decoherence-free if all matrix elements of the form  $\langle \mathbf{n}\varphi' | H_{\text{cond}} | 0\varphi \rangle$  vanish for  $n \neq 0$  and arbitrary  $\varphi'$ . This is the case iff

$$J_- |\varphi\rangle \equiv \sum_{i=1}^N \sigma_i |\varphi\rangle = 0, \quad (18)$$

which defines the trapped states of  $N$  atoms. Atomic states fulfilling this condition are well known in quantum optics as the Dicke states, of the form

$$|\varphi\rangle = |j, -j\rangle \quad (19)$$

in the usual  $|j, m\rangle$  notation with  $j = 0, 1, \dots, N/2$  or  $j = 1/2, 3/2, \dots, N/2 - 1$ , depending on whether  $N$  is even or odd respectively. Here  $j$  is a quantum number with  $n = N/2 - j$ , if  $n$  is the number of excitations in the system. The dimension of all possible trapped states for a fixed number of atoms  $N$  equals therefore [9]

$$\binom{N}{(N+1)/2} \quad \text{or} \quad \binom{N}{N/2} \quad (20)$$

depending on whether  $N$  is odd or even. For large atom numbers the dimension of the decoherence-free subspace equals in a good approximation  $(2/(\pi N))^{1/2} \cdot 2^N$  and thus increases almost exponentially with the atom number  $N$ .

For two atoms there are two independent decoherence-free states,  $|000\rangle$  and  $|0a\rangle$ , as shown in the previous Section. With the help of these two states also the trapped states of  $N$  atoms in the cavity can be constructed. One can show that each state of the form  $|\varphi\rangle = |0\rangle_2 \otimes |a\rangle_{13} \otimes |a\rangle_{45} \otimes \dots \otimes |0\rangle_N$  in which for instance the second atom is in the ground state, the first and third are in an antisymmetric state and so on fulfills condition (18). Writing down all possible states of this form leads to an overcomplete basis which can then be normalised.

### C. Manipulation of the states

We assume that the  $N$  two-level atoms are confined to fixed positions inside the cavity and spatially well separated so that laser pulses can be applied to each atom individually. We denote the complex Rabi frequencies for atom  $i$  by  $\Omega_i$ . In the presence of the laser field the conditional Hamiltonian becomes the sum of  $H_{\text{cond}}$  given in Eq. (16) and the laser Hamiltonian  $H_{\text{Laser I}}$ ,

$$H_{\text{laser I}} = \frac{\hbar}{2} \sum_{i=1}^N \Omega_i \sigma_i + \text{h.c.} \quad (21)$$

In the following we assume that the laser pulse is very weak and that all non vanishing Rabi frequencies  $\Omega_i$  fulfill the condition

$$\Gamma \ll |\Omega_i| \ll \kappa, \quad g^2/\kappa. \quad (22)$$

In this parameter regime spontaneous emission can be neglected in first approximation and we can show that the system, once prepared in a decoherence-free state cannot move out of the subspace. An explanation for this is given below.

In the presence of the laser field the effect of the interaction of the system with its environment over a time interval  $\Delta$  of the order of  $15/\kappa$  and  $15\kappa/g^2$ , can be interpreted as a measurement whether the system is in a decoherence free state or not because  $\Delta t$  is, due to the conditions (5) and (22), much smaller than  $1/|\Omega_i|$ . The effect of the laser during a time interval  $\Delta t$  can be neglected. This underlying process has been discussed in more detail on the example for two atoms in the previous Section.

Here the system continuously interacts with its environment and continuous measurements are performed on the system. Therefore the effect of a weak laser pulse on the system can easily be interpreted with the help of the quantum Zeno effect [18]. The quantum Zeno effect is a consequence of the projection postulate [15] and has been verified experimentally [21]. It predicts a slowing down of the time evolution of a system due to rapidly repeated ideal measurements. In the case of vanishing times between subsequent measurements the system is “frozen” in the state or *subspace of states* it is once found in. Applying this to the system we discuss here, the quantum Zeno effect suggests: If the state of the system belongs initially to the decoherence-free subspace it will remain decoherence-free with a very high probability, even if the laser excites transitions out of the subspace.

But only the transitions out of the decoherence-free subspace are inhibited. The time evolution of the system inside the decoherence-free subspace is not affected and the state of the system at the end of a laser pulse of length  $t$  equals in a very good approximation

$$|\psi(t)\rangle = \mathcal{P}_{\text{DFS}} \dots \mathcal{P}_{\text{DFS}} U_{\text{cond}}(\Delta t, 0) \mathcal{P}_{\text{DFS}} U_{\text{cond}}(\Delta t, 0) |\psi_0\rangle \equiv U_{\text{eff}}(t, 0) |\psi_0\rangle, \quad (23)$$

where  $\mathcal{P}_{\text{DFS}}$  is the projector on the decoherence-free states and  $|\psi_0\rangle$  the initial (decoherence-free) state of the system. The operator  $U_{\text{eff}}$  is an *effective* time evolution operator. Neglecting spontaneous emission ( $\Gamma = 0$ ) leads to the effective Hamiltonian

$$H_{\text{eff}} = \mathbb{P}_{\text{DFS}} H_{\text{laser I}} \mathbb{P}_{\text{DFS}} . \quad (24)$$

This Hamiltonian describes the effect of the weak laser pulse to a very good approximation. Note that it is independent of the specific values of the parameters  $g$  and  $\kappa$ .

The states of the decoherence-free subspace can be used to obtain stable qubits for quantum computing. In addition, the effective Hamiltonian allows to prepare the system in an entangled state and to perform gate operations *inside* the subspace. An example for a possible realisation of a nearly decoherence-free quantum computer is given in Ref. [11]. In Ref. [22] an experimental test of Bell's inequality using atoms is proposed. The scheme is based on the ideas discussed here, but due to the use of an additional level it also works with a very high efficiency if  $\Gamma$  and  $g$  are of the same order of magnitude.

#### D. Example

To check how well the proposed scheme works, let us reconsider the case of two two-level atoms inside the cavity. In this case Eq. (16), (21), and (24) lead to the effective Hamiltonian

$$H_{\text{eff}} = \frac{\hbar}{2\sqrt{2}} \left[ (\Omega_1 - \Omega_2) |000\rangle\langle 0a| + \text{h.c.} \right] , \quad (25)$$

so that to first approximation the laser just leads to oscillations between the two decoherence-free states  $|000\rangle$  and  $|0a\rangle$ . This is in good agreement with Fig. 4 which results from a numerical solution of Eq. (9)–(11). A more detailed analysis of the time evolution of this system can be found in Ref. [9].

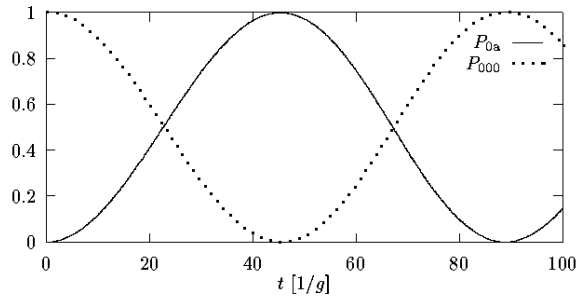


FIG. 4. The time evolution of the population of the decoherence-free states of two atoms in presence of a laser pulse focused on atom 1 derived from numerical solution of the full dynamics of Eq. (9)–(11). The coupling constant  $g$  has been chosen equal to  $\kappa$  and it is  $\Omega_1 = 0.05 g$ ,  $\Omega_2 = -\Omega_1$  and  $\Gamma = 10^{-3} g$ .

By choosing the length of the laser pulse in an appropriate way the scheme can also be used to prepare the atoms in the maximally entangled state  $|a\rangle$ . Fig. 5 shows the success rate  $P_0$  for this scheme and also results from a numerical solution. For zero spontaneous emission, success rates arbitrarily close to unity can be achieved by reducing the Rabi frequency  $\Omega_1$ . However, for  $\Gamma \neq 0$  this is not possible without increasing the probability of occurrence of a spontaneously emitted photon. For finite values of  $\Gamma$  there is an optimal value of  $\Omega_1$  for which the success rate of the preparation scheme has a maximum. The fidelity of the prepared state can, for a very wide parameter regime, be very close to 1 [9].

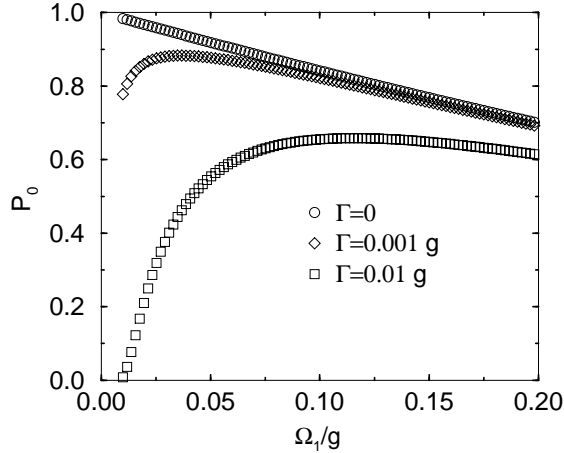


FIG. 5. Probability for successful preparation of the maximally entangled state  $|a\rangle$  as a function of the Rabi frequency  $\Omega_1$  for  $\Omega_2 = -\Omega_1$ ,  $\kappa = g$ , and different values of  $\Gamma$ .

#### IV. TELEPORTATION IMPLEMENTED BY THE DETECTION OF PHOTONS

Another example in which cavity decays can play a constructive role is in the quantum teleportation (see Ref. [23] for teleportation) of an atomic state. Our example derives from earlier examples of entangling distinct atoms by detection (or the non-detection) of decays [7,8]. In all experimental implementations of teleportation to date [24], the stationary qubits have been of transient "flying" optical origin. In earlier proposals of atomic state teleportation [25], the flying qubits were atomic states and are therefore not ideal for long distance teleportation. Our scheme differs from these earlier in using both the ideal stationary (atomic) and the ideal flying (photonic) qubits. It also differs crucially from the much studied quantum communication protocols in which a photon *directly* transfers quantum information from an atom trapped in a cavity to another atom in a distant cavity [26,27]. Our scheme does not require a direct carrier of quantum information between distant atoms. Joint detection of photons *leaking out* of distinct cavities enables *disembodied* transfer of quantum information from an atom in one of the cavities to an atom in the other. We thus provide a quantum state transfer scheme that does not require us to inject a photon into a cavity from outside [26,27]. It should also be made clear that our scheme is very different from establishing prior entanglement between two distant atoms and subsequent teleportation of the unknown state of a third atom using this entanglement. In that case we would require at least two atoms trapped in one cavity. For the basic operation of our scheme, it is sufficient to have only one atom in each cavity. For a failure-proof operation, however, we would require an additional atom in one of the cavities.

The setup consists of two optical cavities, each containing a single trapped  $\Lambda$  three level atom, as shown in Fig.6. Atoms 1 and 2 are trapped in cavities A and B (supporting cavity modes A and B) respectively. The photons leaking out from both the cavities are incident on the 50 – 50 beam splitter  $S$  and are detected at  $D_+$  and  $D_-$ . We assume unit efficiency detectors initially (we include finite efficiency later). The cavity A, atom 1, beam splitter  $S$  and the detectors  $D_+$  and  $D_-$  belong to Alice. The cavity B with atom 2 belongs to Bob. We require both the cavities to be *one sided* so that the only leakage of photons occur through the sides of the cavities facing  $S$ . By following our teleportation protocol, Alice can teleport an unknown state of her atom 1 to the atom 2 held by Bob in three stages.



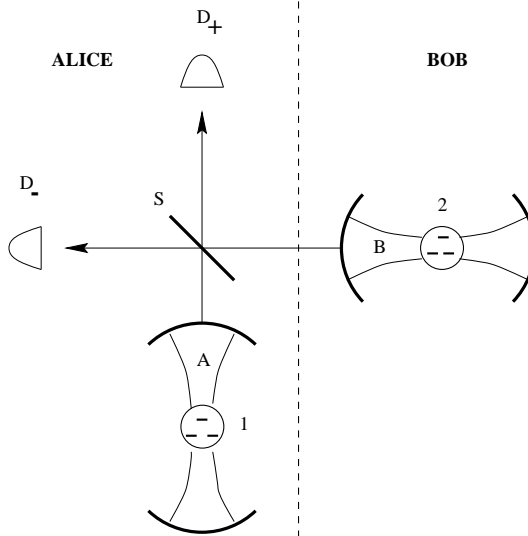


FIG. 6. The atomic state teleportation setup. The cavity A, atom 1, beam splitter  $S$  and the detectors  $D_+$  and  $D_-$  belong to Alice, while the cavity B and atom 2 belong to Bob.

In the preparation stage, Alice maps her atomic state to her cavity state [30]. At the same time Bob creates a maximally entangled state of his atom and his cavity mode. In the next stage (the detection stage) Alice waits for a *finite* time for either or both of her detectors to click. If any one of the detectors register a single click during this time period, then the protocol is successful. Otherwise Alice informs Bob about her failure.

This protocol can be related to the standard teleportation protocol [23] by noting that the beam splitter and the detectors constitute a device for measurement of the joint state of the two cavities in the basis  $\{|0\rangle_A|0\rangle_B, |1\rangle_A|1\rangle_B, \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B), \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)\}$ . Here  $\{|0\rangle_A, |1\rangle_A\}$  and  $\{|0\rangle_B, |1\rangle_B\}$  are photon number states in cavities A and B respectively. The teleportation is probabilistic, because it is successful only for the pair of Bell state outcomes of the above measurement (later we describe how to convert this to a *reliable* state transfer protocol). At the end of the detection period, if the protocol has been successful, Alice lets Bob know whether  $D_+$  or  $D_-$  had clicked. This corresponds to the classical communication part of the standard teleportation protocol [23]. Dependent on this information Bob applies a local unitary operation to his atom to obtain the teleported state, in a post-detection stage.

The detailed analysis of the scheme has been given in Ref. [10], and only the results will be presented here. We looked at single realizations conditioned on detection (or not) of cavity decays using the quantum jump approach [16]. The trapped atoms are considered to be three level atoms as shown in Fig.7.

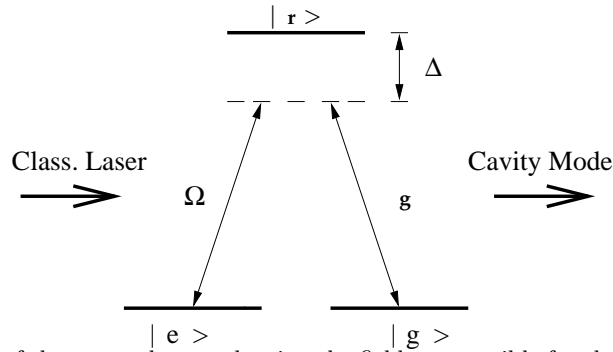


FIG. 7. The level configuration of the trapped atom showing the fields responsible for the adiabatic evolution. The  $|e\rangle \rightarrow |r\rangle$  transition being driven by a classical laser field of coupling  $\Omega$  and the  $|r\rangle \rightarrow |g\rangle$  transition being driven by the quantized cavity mode of coupling  $g$ .  $\Delta$  is the detuning of both the classical laser field and the quantized field mode from their respective transitions.

Alice and Bob use two types of time evolutions of the atom-cavity system as their basic local operations. The first type is an adiabatic evolution (shown in Fig.7) which is initiated by switching on a classical laser field which drives the  $|e\rangle \rightarrow |r\rangle$  transition with a coupling constant  $\Omega$ . The  $|r\rangle \rightarrow |g\rangle$  transition is driven by the quantized cavity mode of coupling  $g$ . Both the classical laser field and the cavity modes are assumed to be detuned from their respective transitions by the same amount  $\Delta$ . As the atom is considered to be trapped at a specific position in the cavity, we

can assume that the couplings  $\Omega$  and  $g$  remain constant during the interaction. We choose parameters such that  $g\Omega/\Delta^2 \ll 1$  (the upper level  $|r\rangle$  can then be decoupled from the evolution) and  $\Delta \gg \gamma$  (the spontaneous decay rate from  $|r\rangle$  can be neglected). The Hamiltonian for the evolution of the system under such conditions (and assuming  $g = \Omega$  for simplicity), is given by  $H^{(1)} = E|e\rangle\langle e| + E|g\rangle\langle g| + E(c|e\rangle\langle g| + c^\dagger|g\rangle\langle e|)$  where  $E = g\Omega/\Delta$  [27] and  $c$  and  $c^\dagger$  are destruction and creation operators for the cavity mode. The other local operation accessible to Alice and Bob is the Zeeman evolution used to give an arbitrary phase shift of the level  $|e\rangle$  relative to the level  $|g\rangle$ . The Hamiltonian for this evolution is  $H^{(2)} = \delta E|e\rangle\langle e|$ , where  $\delta E$  is an energy difference. The decay rate from both cavities is assumed to be the same and equal to  $\kappa$ .

Let the unknown state of the atom 1 which Alice wants to teleport be

$$|\Psi\rangle_1^I = a|e\rangle_1 + b|g\rangle_1, \quad (26)$$

where the superscript  $I$  in  $|\Psi\rangle_1^I$  stands for input and  $a$  and  $b$  are complex amplitudes. We assume that the initial state of Alice's cavity is  $|0\rangle_A$  and the initial state of Bob's atom-cavity system is  $|e\rangle_2|0\rangle_B$ . Alice first maps the state of atom 1 onto the cavity mode A by switching the Hamiltonian  $H^{(1)}$  on for a period of time  $t_I$  given by  $\tan \frac{\Omega_\kappa t_I}{2} = -\frac{\Omega_\kappa}{\kappa}$  where  $\Omega_\kappa = \sqrt{4E^2 - \kappa^2}$ . Subject to no decay being recorded in the detectors, the cavity state is given by

$$|\Psi\rangle_A^I = \frac{1}{\sqrt{|a|^2\alpha^2 + |b|^2}}(a\alpha|1\rangle_A + b|0\rangle_A), \quad (27)$$

where  $\alpha = (\frac{e^{-\frac{\kappa}{2}t_I}}{\Omega_\kappa}2E \sin \frac{\Omega_\kappa t_I}{2})$ . The probability that no photon decay takes place during this evolution is given by  $P_{ND}(A) = (|a|^2\alpha^2 + |b|^2)$ . Meanwhile, Bob also switches on the Hamiltonian  $H^{(1)}$  in his cavity for a shorter length of time  $t_E$  given by  $\tan \frac{\Omega_\kappa t_E}{2} = -\frac{\Omega_\kappa}{2E - \kappa}$ . His atom-cavity system thus evolves to the entangled state

$$|\Psi\rangle_{2,B}^E = \frac{1}{\sqrt{2}}(|e\rangle_2|0\rangle_B + i|g\rangle_2|1\rangle_B). \quad (28)$$

The probability that no photon decay takes place during this evolution is given by  $P_{ND}(B) = |\beta|^2$  where  $\beta = \frac{e^{-\frac{\kappa}{2}t_E}}{\Omega_\kappa}2\sqrt{2}E \sin \frac{\Omega_\kappa t_E}{2}$ . For simplicity, we assume that Alice and Bob synchronize their actions such that the preparation of the states  $|\Psi\rangle_A^I$  and  $|\Psi\rangle_{2,B}^E$  terminate at the same instant of time. Now comes the detection stage, in which Alice simply waits for any one of the detectors  $D_+$  or  $D_-$  to click over a finite detection time denoted by  $t_D$ . Alice and Bob reject the cases in which Alice does not register any click, or when she registers two clicks. In the post detection stage, Bob uses  $H^{(2)}$  to give  $|g\rangle_2$  an extra phase shift with respect to  $|e\rangle_2$ . This phase shift is  $-i$  if  $D_+$  had clicked and  $i$  if  $D_-$  had clicked. This concludes the entire protocol.

The average density matrix of Bob's atom generated through to our teleportation procedure is given by  $\rho_2^{Tel} = \{P_{ND}(A)|\Psi\rangle_2\langle\Psi| + 2|a|^2\alpha^2e^{-2\kappa t_D}|g\rangle_2\langle g|_2\}/\{P_{ND}(A) + 2|a|^2\alpha^2e^{-2\kappa t_D}\}$ , where  $|\Psi\rangle_2 = (a\alpha|e\rangle_2 + b|g\rangle_2)/\sqrt{|a|^2\alpha^2 + |b|^2}$ . The fidelity of this state with respect to the input state is  $F(t_D, a, b) = \{P_{ND}(A)(|a|^2\alpha^2 + |b|^2) + 2|a|^2\alpha^2e^{-2\kappa t_D}|b|^2\}/\{P_{ND}(A) + 2|a|^2\alpha^2e^{-2\kappa t_D}\}$ . We see that apart from the system parameters  $\kappa$  and  $\Omega_\kappa$ , the fidelity of the generated state also depends on the detection time  $t_D$  and the modulus of the amplitudes  $a$  and  $b$  of the initial state. It is a teleportation protocol with a *state dependent fidelity*.

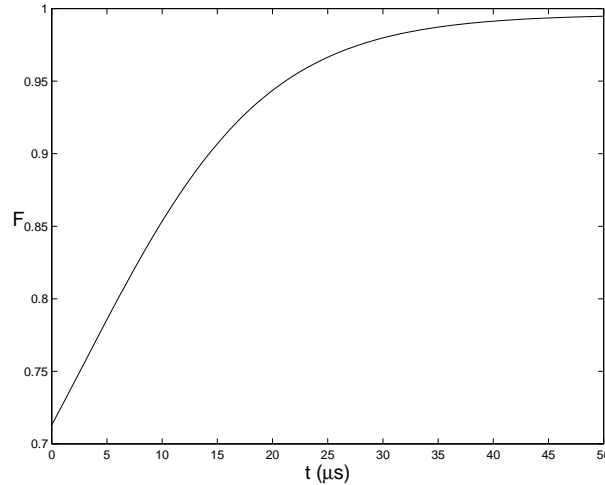


FIG. 8. The improvement of average teleportation fidelity with the length of the detection stage. The parameter regime is  $(g : \Omega : \kappa : \gamma : \Delta)/2\pi = (10 : 10 : 0.01 : 1 : 100)$  MHz

We plot the variation of the average fidelity of teleportation over all possible input states as a function of the detection time  $t_D$  in Fig.8. We see that the fidelity increases with increasing detection time. This happens because increasing the detection time decreases the proportion of  $|g\rangle_2|g\rangle_2$  in the teleported state  $\rho_2^{Tel}$  and brings it closer to the initial state  $|\Psi\rangle_1^I$  of Alice's atom. The parameter regime used for Fig.8  $\{(g : \Omega : \kappa : \gamma : \Delta)/2\pi = (10 : 10 : 0.01 : 1 : 100) \text{ MHz}\}$ , is carefully chosen to satisfy all our constraints ( $g\Omega/\Delta^2 \ll 1, \Delta \gg \gamma, \Omega_\kappa \gg \kappa$ ). This regime could be approached, for example, by increasing the cavity finesse of Ref. [28] by an order of magnitude and increasing the length of that cavity to about a millimeter while keeping the beam waist and other parameters constant. As evident from Fig.8, the average fidelity exceeds 0.99 for a detection time of about half the cavity life time. For detectors of finite efficiency  $\eta$ , the fidelity of the teleported state would be  $\{\eta P_{1D}(0, t_D)F(t_D, a, b) + 2\eta(1 - \eta)(1 - P_{ND}(0, t_D) - P_{1D}(0, t_D))|b|^2\}/P_{\text{suc}}(\eta)$ , where  $P_{ND}(0, t_D)$  is the probability of no decay during the detection period. For a  $\eta$  of 0.6 and detection times large compared to the cavity decay time, the fidelity of the state in Bob's cavity becomes  $\sim 0.81$ .

The above probabilistic teleportation protocol can be modified to *teleportation with insurance*, so that in the cases when the protocol is unsuccessful, the original state of Alice's atom 1 is not destroyed, but mapped onto another reserve atom  $r$  trapped in Alice's cavity. To accomplish this, Alice has to follow the *local redundant encoding* of Ref. [29] and codes her initial state  $|\Psi\rangle_1^I$  as  $a(|e\rangle_1|g\rangle_r + |g\rangle_1|e\rangle_r) + b(|g\rangle_1|g\rangle_r + |e\rangle_1|e\rangle_r)$ . After this, she just follows the same protocol as before. But in cases when the protocol is unsuccessful, she is left with either the state  $a|g\rangle_r + b|e\rangle_r$  or a state that can be converted to  $a|g\rangle_r + b|e\rangle_r$  by a known unitary transformation. She can now exchange the roles of atom 1 and atom  $r$  and try to teleport the state  $|\Psi\rangle_1^I$  again. She can repeat this procedure until teleportation is successful (Of course, this holds true perfectly only when  $\eta = 1$ ).

In this section, we have shown how the state of an atom trapped in a cavity can be teleported to a second atom trapped in a distant cavity simply by detecting photon decays from the cavities. This is a rare example of a decay mechanism playing a constructive role in quantum information processing. The scheme is comparatively easy to implement, requiring only the ability to trap a single three level atom in a cavity. Moreover, by adding one more atom to Alice's cavity, it can be converted to a *reliable* state transfer protocol. This state transfer protocol can be viewed as an *alternative* to designer laser pulses for transferring (Refs. [26,27]) quantum information into a cavity from outside.

## V. CONCLUSIONS

Summarising, we discussed in this paper how the interaction of a system with its environment can be used to produce entanglement in a controlled way. As an example we discussed the preparation and manipulation of entangled states of atoms inside a cavity and showed how quantum communication protocols such as teleportation can be implemented by the detection of photon decay from cavities containing trapped ions or atoms.

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